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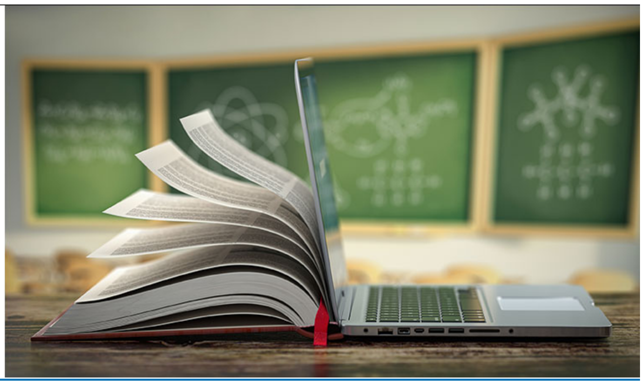

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Research impact of queue choosing policy on macroscopic characteristics in multichannel non-stationary queue system

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Abstract. A multichannel non-stationary queuing system model of stadium checkpoint system studied. Input flow of model is based on statistical information of visitors flows from football matches in the Russian Federation. An algorithm choosing the shortest queue for implementing a policy of choosing a non-random queue presented. Result of comparing different policies described for input rate like real ones. The result is substantiated by comparing the functioning of the stationary QS with different queue selection policies in 3 modes: underload, load, overload.

1. Introduction

Multichannel non-stationary queuing systems (MNQS) are used today as mathematical models to describe various technical systems [1][2], including: Internet banks, at the time of mass payment of wages, equipment of mobile operators, passenger control devices at airports and at railway stations, information control systems (ICS) of facilities for holding public events, etc.

The study of the features ICS functioning is of undoubted interest from a practical point of view. Because the results obtained in this case can be used as a scientific justification for the design decisions taken at the stage of their design and modernization, as well as in the development of measures to ensure safety during mass event. One of the features of the system under discussion is that a visitor (in terms of the QS theory it is client) has the ability to choose a turnstile based on his own preferences (for example, choose a turnstile (in terms of the QS it is server), with the shortest queue length. Both stationary QS and non-stationary QS studies consider in most cases random choice of the queue. That is why, the study of the impact of the rule for choosing a queue by the client (server choosing policy) is a relevant task.

The article presents the results of a study impact server choosing policy on the dynamics of the MQS and MNQS.

2. Mathematical model of input rate and service rate

Figure 1 shows dependences of $\frac{\lambda_{\exp}}{\max(\lambda_{\exp})}(t)$ based on statistics analysis. Statistics was collected during 18 football matches held in Yekaterinburg, St. Petersburg, Samara.

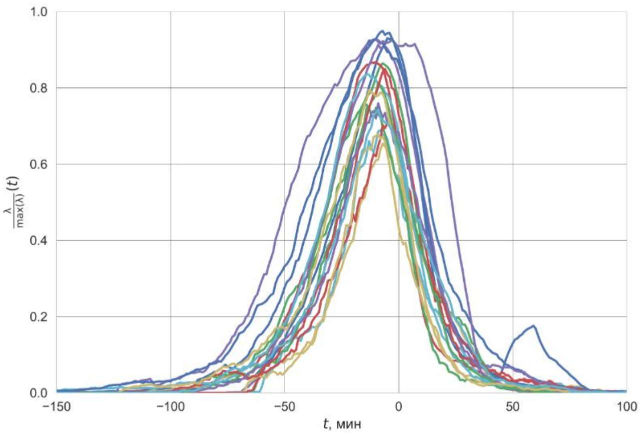

Figure 1. Dependency $\frac{\lambda_{\exp}}{\max(\lambda_{\exp})}(t)$ graphs.

Figure 1 shows the main features of the law of change in the deterministic component of the input rate of clients. $\lambda(t)$ first monotonically increases from zero to a certain maximum value $\lambda_{\max}$ on the time interval $[-120;-20]$ minutes (time $t=0$ corresponds to the beginning of the match event). After $\lambda(t)$ monotonically decreases in the time interval $[-20;20]$ minutes to zero.

In [3] the possibility of use piecewise constant approximation of $\lambda(t)$ for NQS studied. The input rate in that paper was the same as previously described dependency $\lambda(t)$:

$$\lambda(t)=\sum_{k=0}^{K}\left(\theta\left(t_k-t\right)-\theta\left(t_{k+1}-t\right)\right)\cdot\bar{\lambda}_k, \tag{1}$$

there
$\theta(t-\xi)$ is Heaviside step function:

$$\theta(t-\xi)=\begin{cases}0, t<\xi,\\ 1, t\geq\xi,\end{cases} \tag{2}$$

$\bar{\lambda}_k$ is mean value of function $\lambda_{\exp}(t)$ on $[t_k, t_{k+1}]$ time interval.

Such approximation means that for each time interval $[\tau_k, \tau_{k+1}]$ studied system could be considered stationary QS. The clients flow is stationary during that interval and input rate is constant λ_k. The initial state of the system corresponds to the final state of the QS on the previous interval $[\tau_{k-2}, \tau_{k-1}]$. In this study, time intervals of 0.5 minutes were used, following [4].

In the studied NQS, clients are served in accordance with the FIFO policy (firs in fist out). The service rate $\bar{\mu}$ of incoming clients is determined by the service time. Service time is a random variable with a probability density $p\{\xi\}$:

$$p\{\xi\}=\begin{cases} 0, \text{ when } \xi<1, \\ \dfrac{2M[\xi]}{9(M[\xi]-1)}(\xi-1), \text{ when } 1\le\xi<M[\xi], \\ \dfrac{2M[\xi]}{9(M[\xi]-10)}(\xi-10), \text{ when } M[\xi]<\xi\le10, \\ 0, \text{ when } \xi>10, \end{cases} \tag{3}$$

and $\xi\in[1,10]$.

We used random numbers generated in accordance with the probability density (3) and $M[\xi]=4$, $\overline{\mu}=15$ in carried research.

Previously in [3] it was proved that the dynamics of the NQS of the chosen type can be described by a set of macroscopic characteristics that do not depend on time. Maximum queue length $L_{\max}$, points in time at which the maximum queue length is reached $t_{L_{\max}}$, maximum waiting time in line $\tau^{w}_{\max}$, points in time at which the maximum waiting time in the queue is reached $t_{\tau^{w}_{\max}}$, number of people entered at the beginning of the match event N_0, points in time at which 97% of all clients will be served T_{All} are such macroscopic characteristics.

It is known from everyday experience that common visitor of a mass event from two adjacent turnstiles (in terms of the theory of QS it is server) chooses the turnstile with the minimal queue of visitors waiting for service. Macroscopic characteristics of the NQS are expected to depend not only from service devices and their quantity, but also on the line selection policy. According to this hypothesis impact of the policy of choosing an server with a minimum queue length on the macroscopic characteristics of NQS was studied.

The block diagram of algorithm is represented in [5]. To simulate multichannel NQS with a randomly selected server, a modified algorithm was used. Dimension is added to all collections to store parameters of several servers. In block 4, a random selection of queue from all of them is added for algorithm #1.

To simulate the multichannel NQS with the selection of the shortest queue to the server was used algorithm (Algorithm #2) that differs from Algorithm #1 by the modified block 4.5. In figure 2 described how the shortest queue is selected.

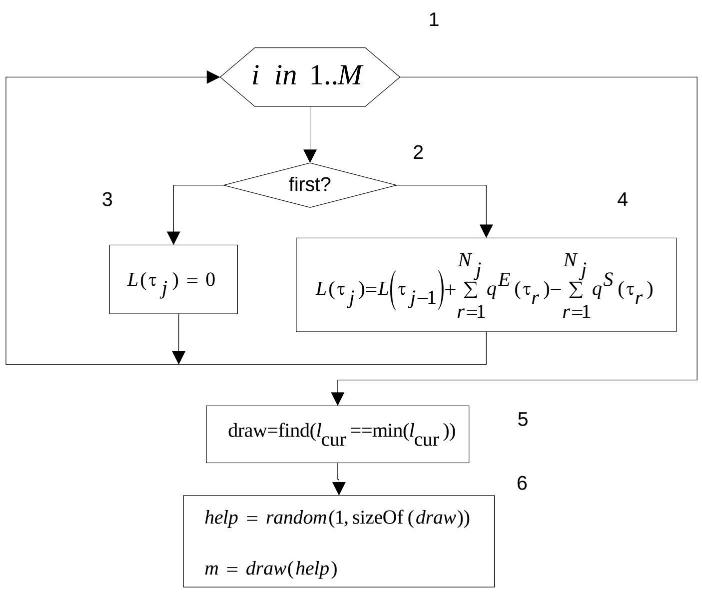

Figure 2. Algorithm for determining the smallest queue.

Figure 2 shows:

- For each server if the client for the given server is the first, the queue length to server is assignment zero, otherwise, the queue length is calculated by the formula of the instantaneous queue length.
- Servers with the minimum queue length are selected from all of servers. Auxiliary array "draw"(draw) of the servers addresses with the minimum queue are forms (block 5).
- In the "draw" by randomly choosing a natural number from the interval [1, sizeOf(draw)] is selected the address help_ind (index in the «draw»). The address m (index in the collections) of the service device selected from the "draw" at the address help_ind is returning (block 6).

3. Computational experiments methods

The dependence $\lambda(t)$ used in these experiments is like that shown in figure 1. Function $\lambda(t)$ on time interval $[-120;-20]$ min monotonically increases from 0 to $\lambda_{\max}$ =35 clients/min, then on the time interval $[-20;30]$ min monotonically decreases from $\lambda_{\max}$ to 0 clients/min. The dependency $\lambda(t)$ provided the service in a two-channel NQS for 1700 clients at each server (a simulation time of ≈200 minutes). In this case, a piecewise constant approximation (by sequence $\lambda_k^{\det}$, $i=\overline{1,1360}$) of the dependence of clients input flow deterministic component $\lambda_{\det}(t)$ from time was used. Service rate is based on a triangular distribution law with $M[\xi]=4$ s. The number of experiments in the Monte Carlo method was *R*=500.

4. Results

The results of calculating the selected macroscopic characteristics are presented in Table 1.

Table 1. NQS's macroscopic characteristics distribution quantile values for different queuing policies.

	server #	Algorithm #1			Algorithm #2		
		0.05	0.5	0.95	0.05	0.5	0.95
$L_{\max}$	1	231	294.5	361.2	238.6	290.1	341.1
	2	227	297	363	238.6	290.1	341.2
$t_{L_{\max}}$	1	0	9.2	20.1	0.1	9.3	20.1
	2	0	9.2	20.1	0	9	19.3
$\tau_{\max}^{w}$	1	19.2	24.5	30	20.5	24.4	28.6
	2	18.9	24.7	30.4	20.5	24.4	28.6
$t_{\tau_{\max}^{w}}$	1	30.9	39.4	47.1	32.5	39.4	46.2
	2	30.9	39.4	47.1	32.5	39.4	46.2
N_0	–	2207	2261.2	2317.9	2213.6	2270.4	2327.8
T_{All}	–	39.2	43.4	47.8	38.4	42.7	47

Table 4 shows that the following macroscopic characteristics of the single server: points in time at which the maximum queue length is reached $t_{L_{\max}}$, maximum waiting time in line $\tau_{\max}^{w}$, points in time at which the maximum waiting time in the queue is reached $t_{\tau_{\max}^{w}}$ and also macroscopic characteristics of all multichannel NQS: number of people entered at the beginning of the match event N_0, points in time at which 97% of all clients will be served T_{All} regardless of the policy of choosing the server turned out to be close to each other.

Thus, the policy of choosing the server in MNQS with the smallest queue length does not have a noticeable effect on the selected macroscopic characteristics.

A two-channel stationary QS was simulated in three different modes to explain this result.

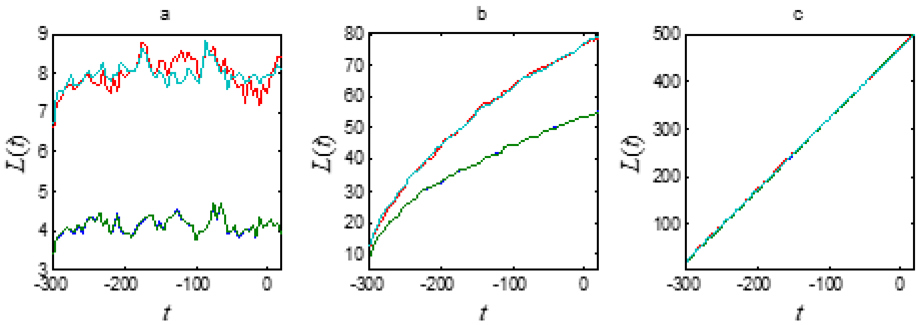

Figure 3. Dependence of the queue length from time for a 2-channel QS in various modes a) underload $\rho_{1,2}=\rho/2=0.9$ b) nearly load $\rho_{1,2}=\rho/2=1$ c) overload $\rho_{1,2}=\rho/2=1.2$.

Figure 3a shows the dependence of the instantaneous queue length in the mode of average channel underload $\rho_{1,2}=\dfrac{\lambda}{\mu\cdot m}=\dfrac{\rho}{2}=0.9$. The policy of choosing a smaller queue leads to a smaller instantaneous queue length, while the queue does not grow. Figure 3b shows the dependence of the instantaneous queue length in the average channel load mode $\rho_{1,2}=\dfrac{\lambda}{\mu\cdot m}=\dfrac{\rho}{2}=1.0$. In this case the policy of choosing a smaller queue results in a smaller instantaneous queue length, but the queue grows infinitely. Figure 3c shows the dependence of the instantaneous queue length in the average channel overload mode $\rho_{1,2}=\dfrac{\lambda}{\mu\cdot m}=\dfrac{\rho}{2}=1.2$. The policy of choosing the smallest queue does not lead to a shorter instantaneous queue length, and the queue grows infinitely for any policy of choosing a queue in such mode.

5. Conclusion

At conclusion if the system has been operating in an overloaded state for a considerable time, then the queue selection policy does not affect the macroscopic quantitative characteristics.

Acknowledgments

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